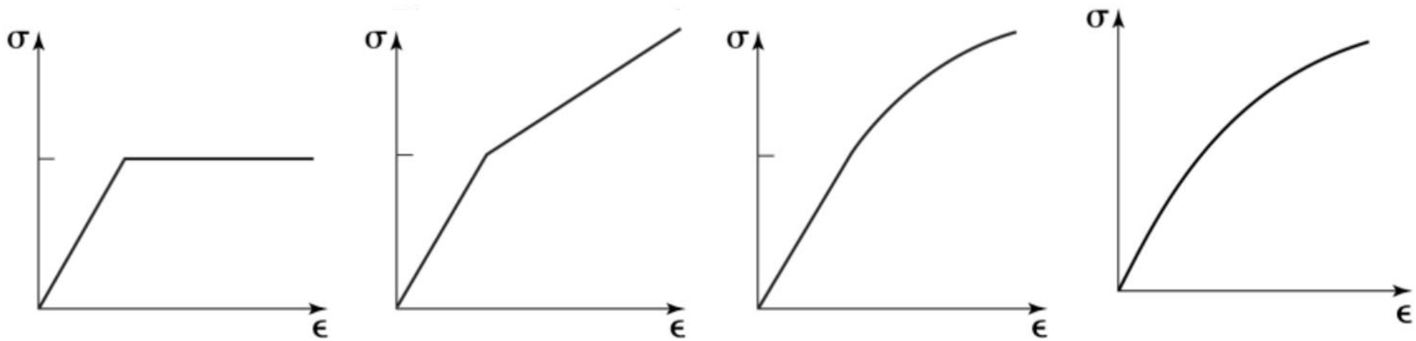
Plasticity and Deformation Process

Non-linear deformation and the effects of deformation process variables

Plasticity is the study of the effect of various nonlinear stress-strain behaviors of materials on their response during processing and operation performance

Often materials exhibit linear stress-strain behavior up to the yield point.

Many commonly used engineering materials exhibit nonlinear stress-strain behavior to various degrees:



a. Elastic – Perfectly Plastic b. Linear Hardening c. Linear-Nonlinear c. Generally Nonlinear

Some materials have inherently non-linear stress-strain relationships

Most plastics and metals are non-linear or plastic

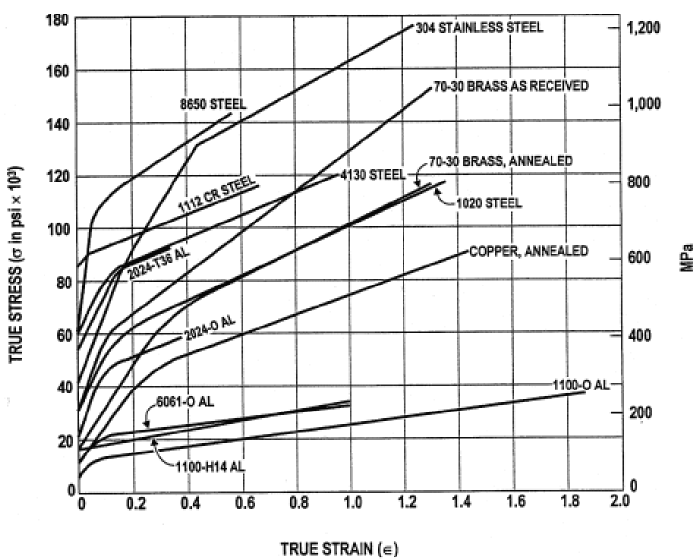
Polymers almost never have linear stress-strain relationship

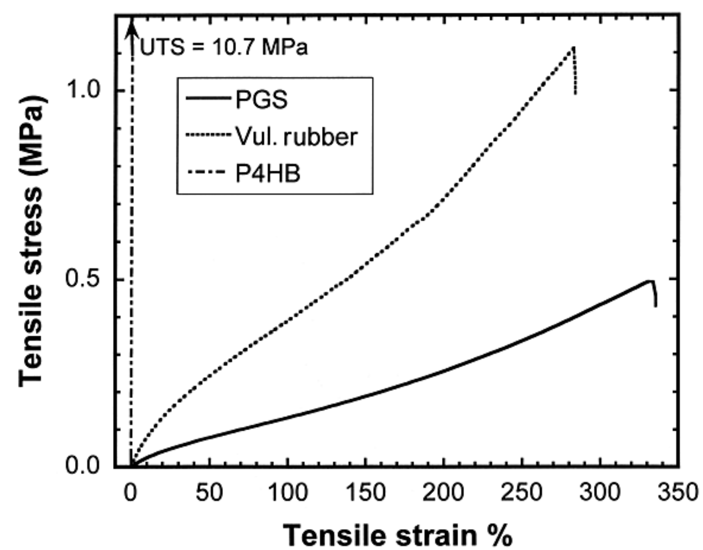
Aluminum is a plastic metal with a very pronounced curvature of the stress-strain curve. It is rarely deformed at low stresses in the linear-elastic range

Rubber is also inherently nonlinear. It is used in bearings and various other load-bearing applications

Concrete has a nonlinear behavior in compression. It deforms in a very small linear elastic range in tension and then fractures in a brittle manner.

Composites like glass fiber in epoxy has some level of non-linear stress-strain behavior due to the nonlinearity of epoxy.



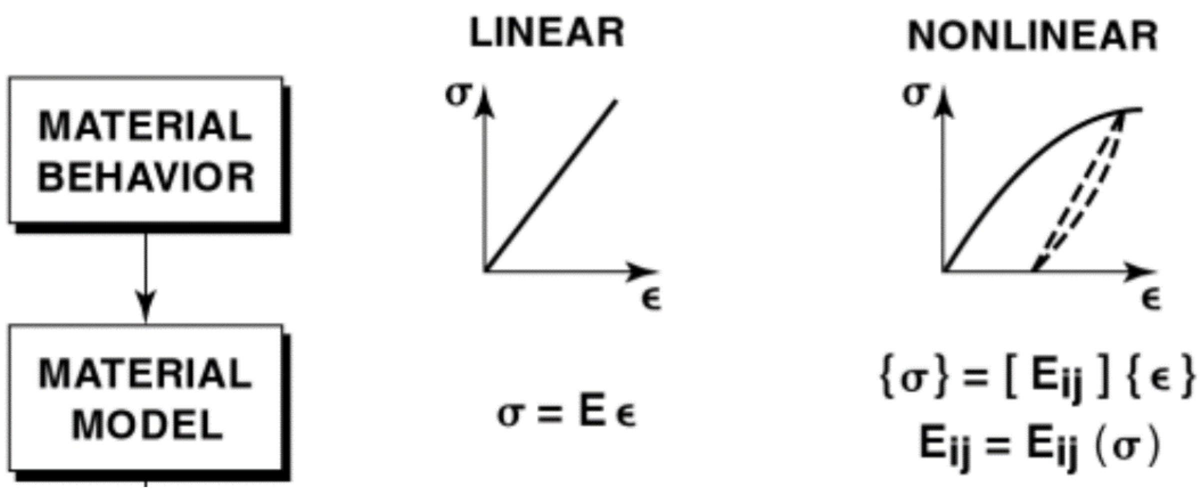


When we consider a material that deforms nonlinearly, a more complicated stress-strain relation is used

Its nonlinearity and the possibility of unloading in various manners lead to not having the same kind of equation that describes the simple linear stress-strain behavior

The Young's modulus is constant for linear elastic deformation

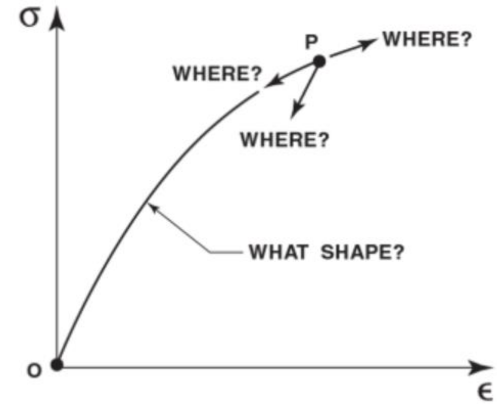
For nonlinear deformation, modulus is a function of the stress level



General nonlinear behavior of engineering materials can be explained by evaluating properties like ductility, yielding, energy absorption, unloading behavior, elevated temperature behavior, bimodular behavior, anisotropy

The resultant characterization of the material enables us find answers to the following questions:

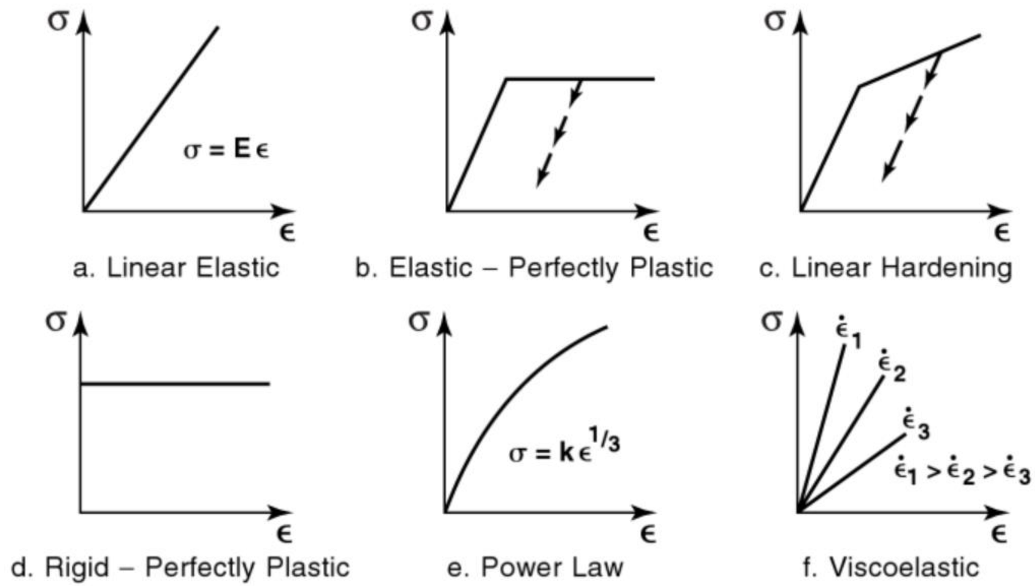
What is the shape of the stress-strain curve upon loading?
In what manner does unloading occur?
In what manner does reloading occur?



The stress-strain curve shape is generally a function of the material, temperature, loading rate, and many other factors that we will consider

Our aim is to develop a material model that can be utilized in calculation of the dynamic modulus and plastic strains

Many idealizations are possible to model the stress-strain behavior of engineering materials

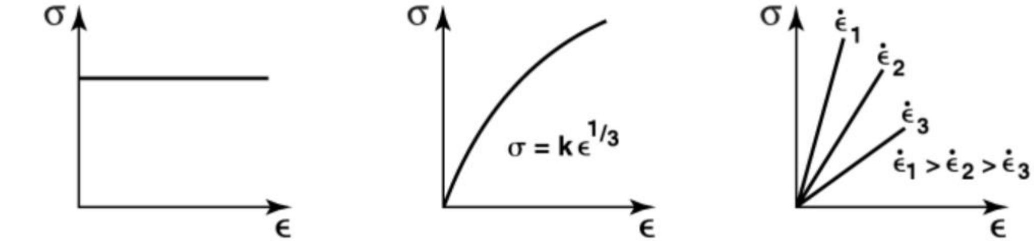


The simplest is the linear elastic approach in which only two material parameters E and ν are needed for the model

The more complex stress-strain behaviors of inelastic materials require more modelling parameters

Elastic-perfectly plastic model is a good approximation to many plastic materials. At least three parameters are required to describe this behavior (E, ν, σ_y)

A linear hardening material has a bilinear modulus in only tension or in only compression
The yield stress increases after the unloading of the material. Hence this type of behavior is observed in loaded-unloaded materials.
At least four parameters are required to describe the deformations.



d. Rigid – Perfectly Plastic e. Power Law f. Viscoelastic

The behavior of some materials in the initial elastic range up to the yield point can be neglected.

The **rigid-perfectly plastic model** provides an accurate representation of the real behavior only if the deformation associated with plastic yielding is much greater than the deformation before yielding.

This is the case in some metal forming processes where the plastic deformation is more than 20x the elastic deformation

This idealization reduces the number of material parameters to only σ_y

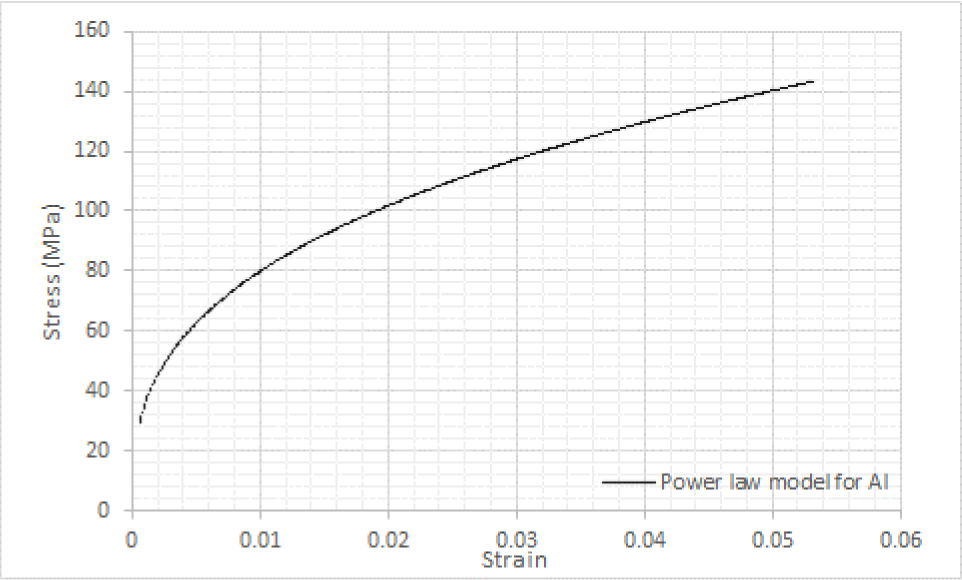
Power-law plastic model represents some materials with large curvature from the start of loading.

The number of material parameters required to describe the deformation behavior depends on the equation. However this is the simplest model used to describe nonlinear behavior

A material may exhibit different deformations under different strain rates: linear for **viscoelastic**, nonlinear for viscoplastic

A viscoplastic polymer can be strained at room temperature at different rates very easily and exhibit different stress-strain behavior in the laboratory conditions.
Plastic aluminum on the other hand, must be subjected to extreme strain rates to exhibit different stress-strain responses to different strain rates.

The stress-strain behavior of aluminum is approximated by the power law equation:
$\sigma = 500 * \varepsilon^{0.5}$



The stress-strain behavior of mild steel is similar to the elastic-plastic idealization

The main difference is that the curve rises to a higher level of stress after yielding than its yield point

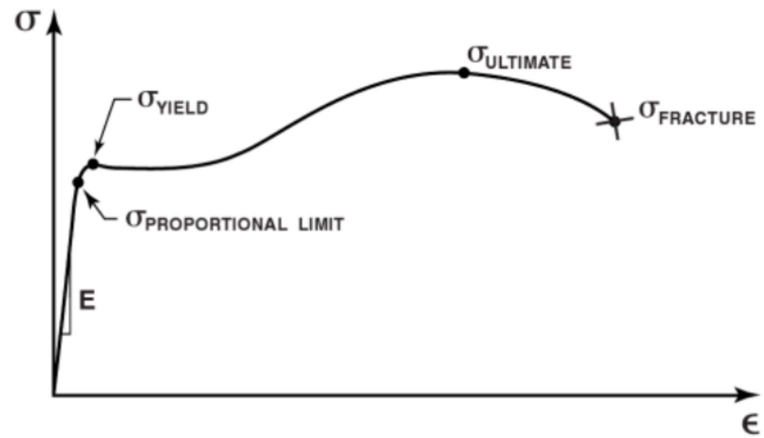


Figure 1-3 Stress-Strain Behavior of Mild Steel

The curve falls off after the ultimate stress from the engineering stress point of view. In that case the large deformations that occur when material necks are not accounted for.

The amount of strain that is obtained after yielding is about 20-30 times the amount of elastic strain.

The deformation behavior of the material can be simplified by considering it to deform linearly until yield point and at constant stress later.

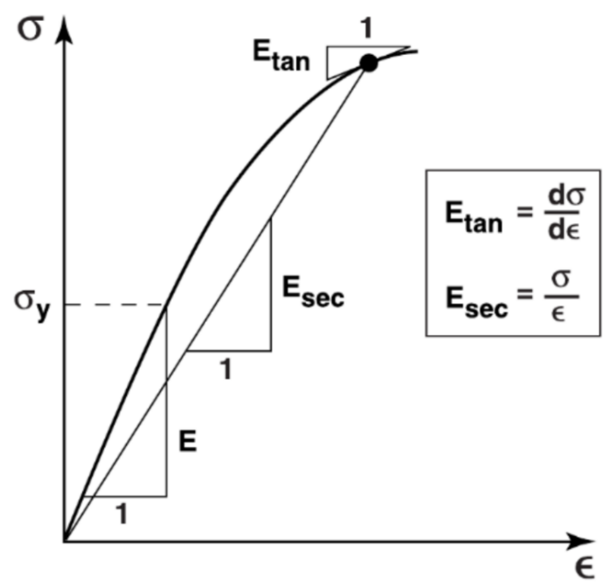
When we describe the stress-strain curve of mild steel in equation form, we use one equation up to the proportional or elastic limit and then use another form of equation.

The nonlinear portion of the stress-strain curve is more difficult to fit into an equation because the modulus is a function of applied stress

Consider a specific point on the nonlinear portion of the curve where we can draw a line back to the origin, the slope of which is called the secant modulus (E_{sec})

The slope of the curve at that point is called the tangent modulus (E_{tan})

The initial Young's modulus is always greater than the secant modulus which is always greater than the tangent modulus. In the linear range all three are the same.



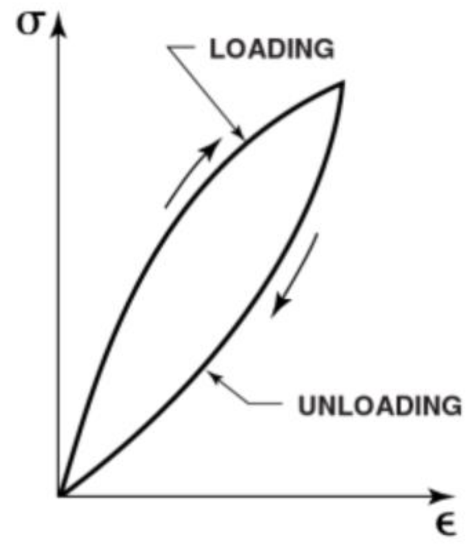
The secant modulus is just a representation that if we know the nonlinear stress level and the modulus, then we can calculate the strain

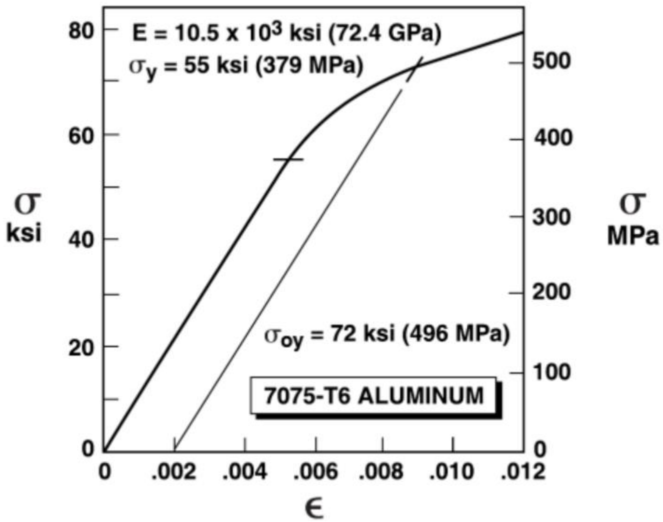
To write an expression for the secant modulus as a function of the stress level is the first part of the solution to the problem of calculating nonlinear strains.

Secant and tangent moduli are used only in loading situations.

A secant modulus during unloading would mean that the material is anelastic (loading and unloading returns the material to the same shape while dissipating the applied energy)

The unloading typically occurs along the appropriate line with the slope of the initial Young's modulus. A permanent set should occur due to the applied energy when zero stress is reached



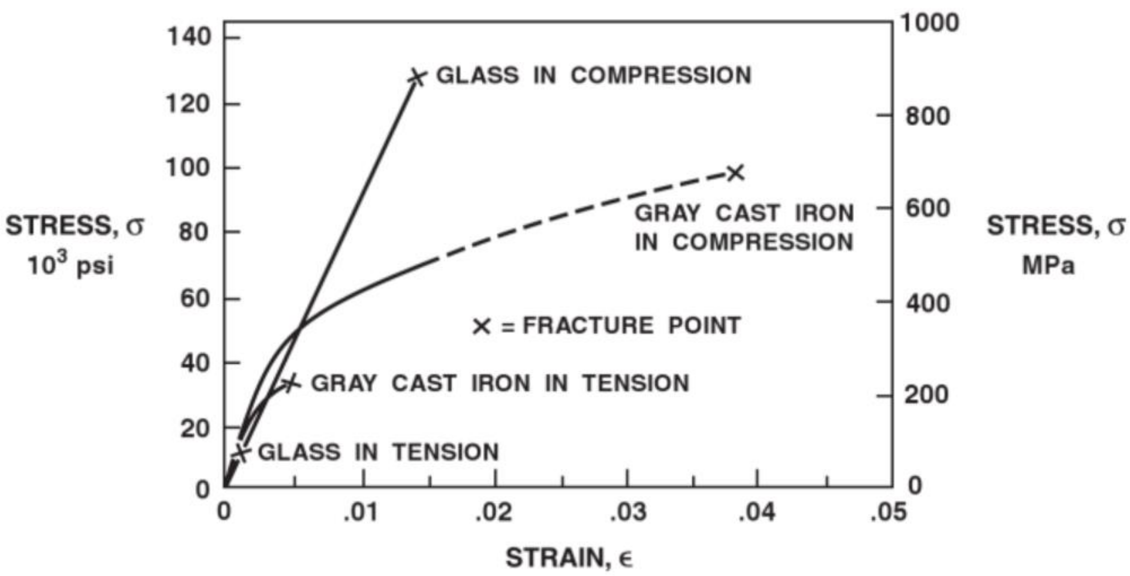


The ductility or brittle behavior of a material depends on the mode of loading: tension or compression

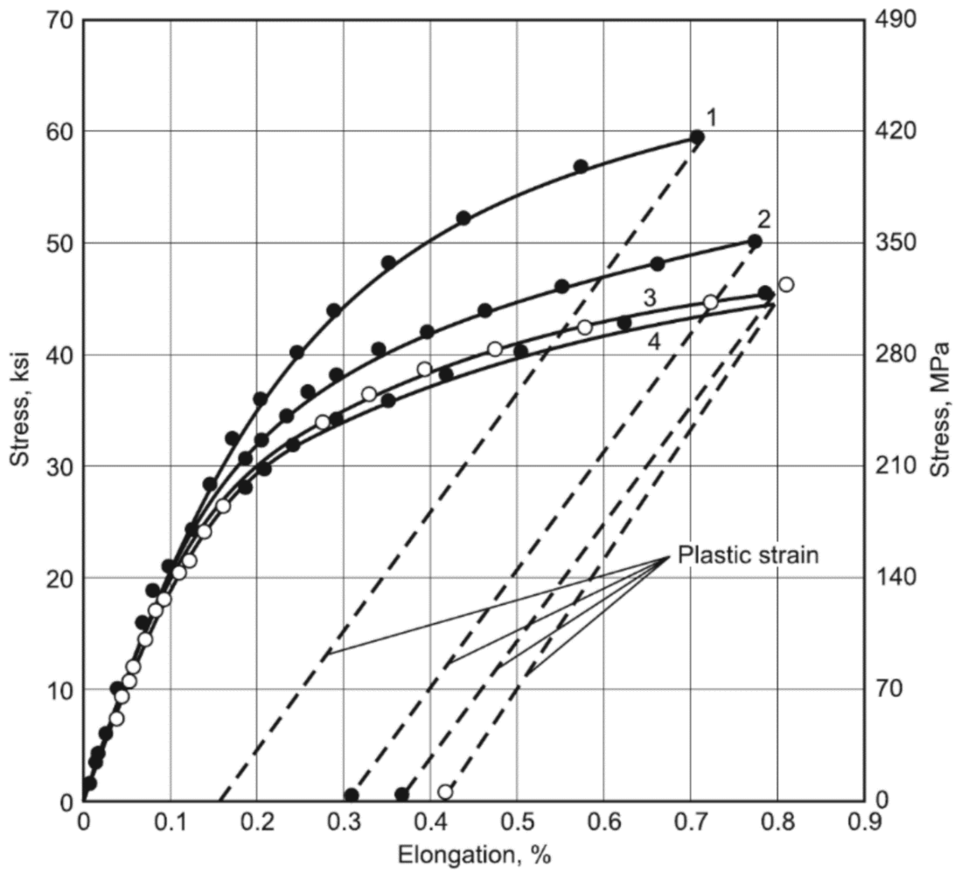
A linear elastic material in both tension and compression is glass which consumes much stress when compressed

On the other hand gray cast iron is brittle under tension and ductile under compression. We have to specify the type of loading to call it either a brittle of ductile material.

As a result the material model and the equation that would represent the stress-strain curve would differ under different types of loading.



Also the shape and size of the structure that we consider affects the material behavior from one part of the structure to another.
For example a beam has a tension region and a compression region.



CI.043 Pearlitic gray iron casting, stress-strain curves showing effect of section size

Casting thickness: curve 1, 12.7 mm (0.5 in.); curve 2, 25.4 mm (1 in.); curve 3, 152.4 mm (6 in.); curve 4, 76.2 mm (3 in.). Dashed lines indicate plastic strain.

Source: C.F. Walton, *Gray and Ductile Iron Castings Handbook,* Gray and Ductile Iron Founders' Society, Aug 1971. As published in *Structural Alloys Handbook,* Vol 1, CINDAS/Purdue University, 1994, p 20

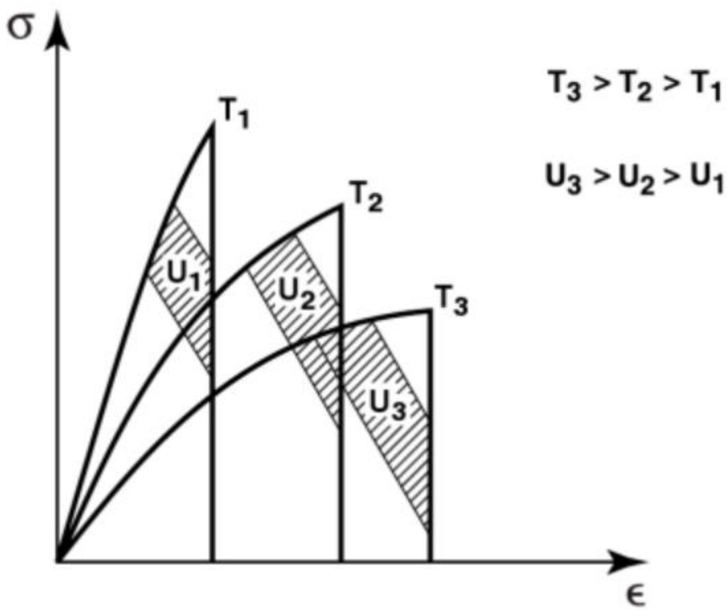
The strain energy density or resilience of a material is a more quantitative analysis than ductile failure.

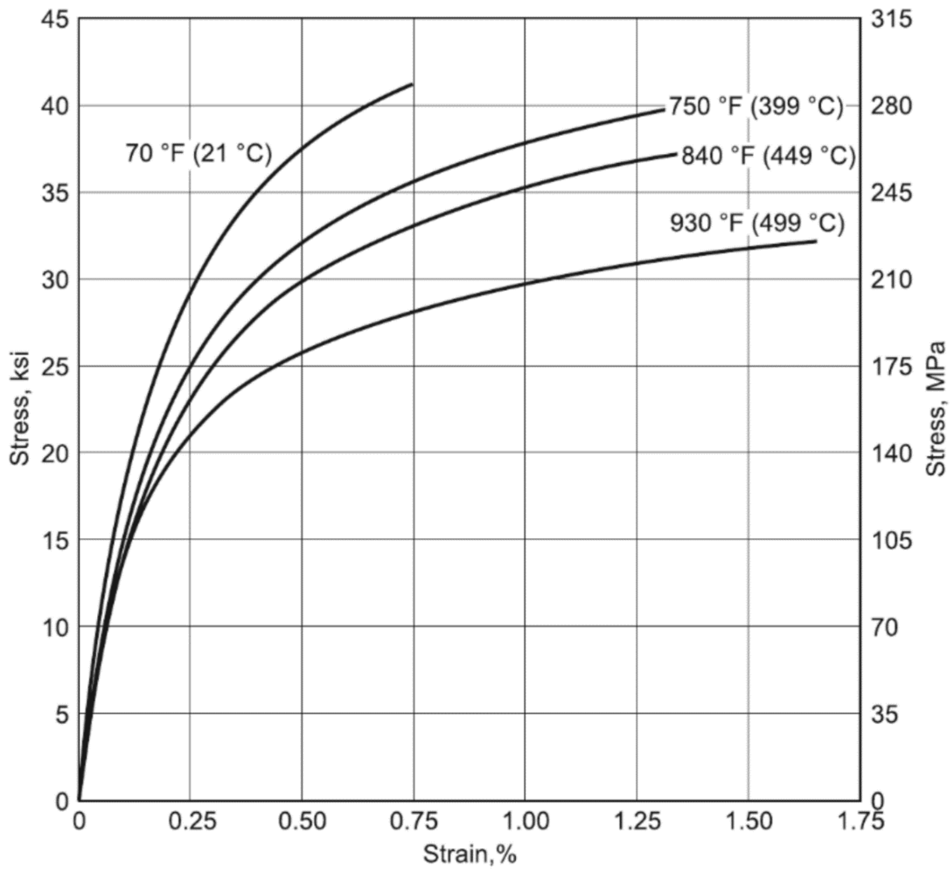
The area under the stress-strain curve (strain energy absorbed per unit volume) provides a number that can be used for comparative purposes

The application of heat and increasing of temperature makes all the stress-strain curves more nonlinear. Decreasing temperature makes materials more linear.

As a consequence of the increased nonlinearity, the material consumes more strain energy as the temperature increases.

Of three different temperatures, which energy is highest depends on the strain rate

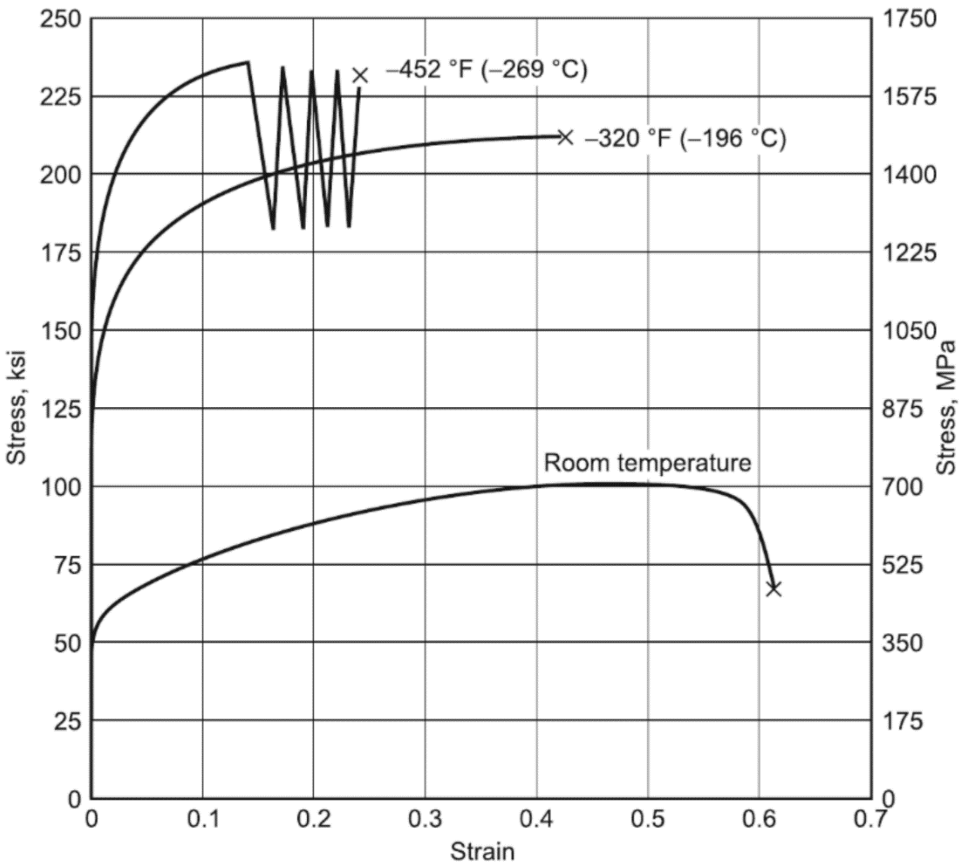




CI.042 Gray iron casting, stress-strain curves to fracture at room and elevated temperatures

Composition: Fe-3.19C-(CC-0.85)-1.66Si- 0.91Mn-0.077P-0.089S

Source: C.F. Walton, *Gray and Ductile Iron Castings Handbook,* Gray and Ductile Iron Founders' Society, 1965. As published in *Structural Alloys Handbook,* Vol 1, CINDAS/Purdue University, 1994, p 20



SS.009 21-6-9 stainless steel, stress-strain curves at room and low temperatures

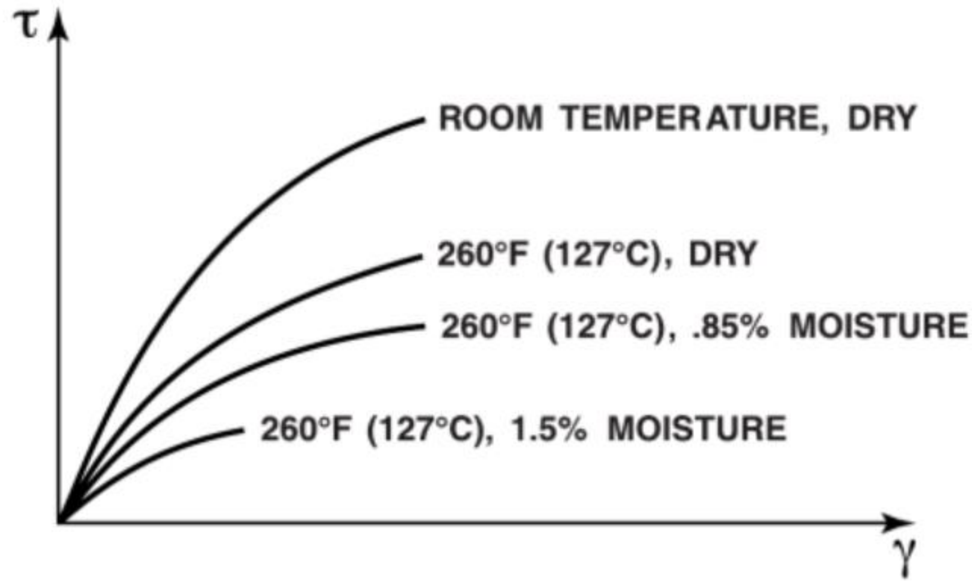
Composition: Fe-20.25Cr-9Mn-6.5Ni-0.28N. UNS S21900

Source: M.B. Kasen, R.E. Schramm, and D.T. Read, "Semi-Annual Report of Materials Research in Support of Super Conducting Machinery," ARPA Order-2569, AD-B063554, National Bureau of Standards, Cryogenics Division, Boulder, CO, Oct 1976. As published in *Structural Alloys Handbook,* Vol 2, CINDAS/USAF CRDA Handbooks Operation, Purdue University, 1994, p 50

Moisture has an influence on material nonlinearity similar to that of temperature for some materials.

The deformation behavior of polymeric matrix materials in fiber reinforced composites is an example.

Both increasing temperature and moisture disturb the material microstructure such that the molecular mobility is increased.



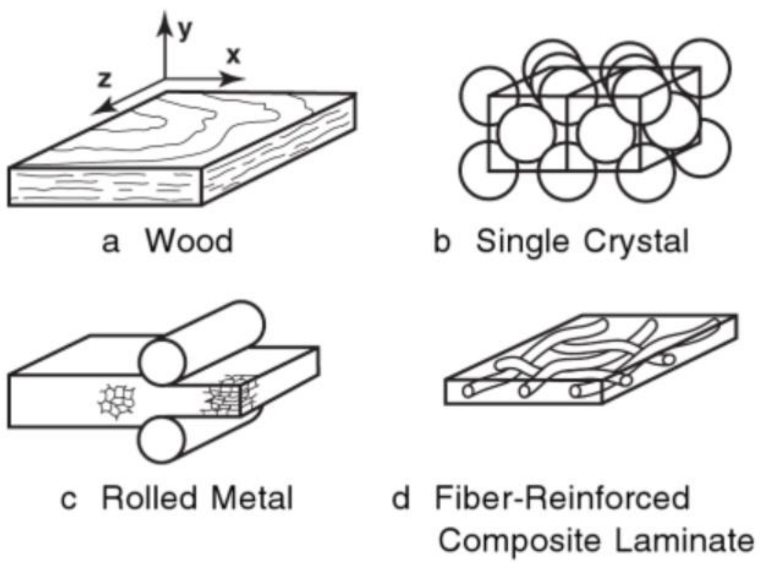
Isotropy means that the material stiffnesses are identical in all directions at a point.
For an anisotropic material a different deformation behavior will be observed from every different direction.

This different deformation behavior may result in various nonlinearities, different Young's moduli or different strengths at different directions.

Transverse isotropy is a simpler anisotropy where the deformation behavior is the same in all directions of one plane but different perpendicular or transverse to that plane.

Orthotropy means the material undergoes different deformations at perpendicular x-y-z directions but some symmetry exists within these planes.

Wood is the most common orthotropic material. Fiber reinforced polymer matrix composite is another.
A crystal with some structural symmetries can be rolled to a thinner, wider material to reorient its structure and create an orthotropic material.



Anisotropic crystalline structure can be visualized as having a shifted set of molecules so that the perpendicular symmetry planes no longer exist.

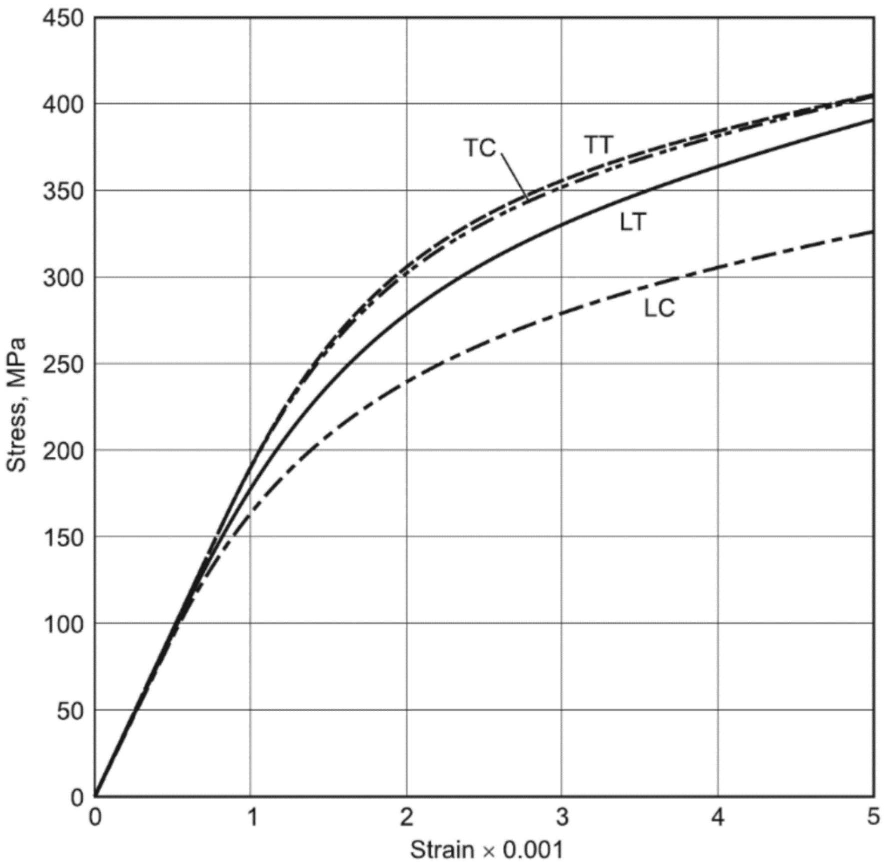
Anisotropic materials have more complicated deformation behaviors due to lack of symmetry.

Table 2-1 Young's Moduli for Orthotropic Materials in Sheet or Lamina Form

Material	E at θ*, 10^6 psi (GPa)		
	0°	45°	90°
Cold-Rolled Iron [2-8]	32.8 (226)	29.3 (202)	39.1 (270)
Cold-Rolled Copper [2-9]	19.8 (137)	15.5 (107)	20.0 (138)
Recrystallized Cold-Rolled Copper [2-9]	10.0 (68.9)	17.5 (121)	9.5 (65.5)
Unidirectional Fiberglass-Epoxy Lamina [2-10]	6.0 (41.0)	1.7 (11.8)	1.5 (10.4)
Bidirectional Fiberglass-Epoxy Lamina [2-10]	3.55 (24.5)	2.02 (14.1)	3.45 (23.8)

*From the direction of rolling for metals and from the 0° direction for fiberglass-epoxy

The higher Young's moduli in the fiber directions of the composite (the lower deformation) reflect the strength of the fibers



SS.002 201 stainless steel sheet, tensile and compressive stress-strain curves

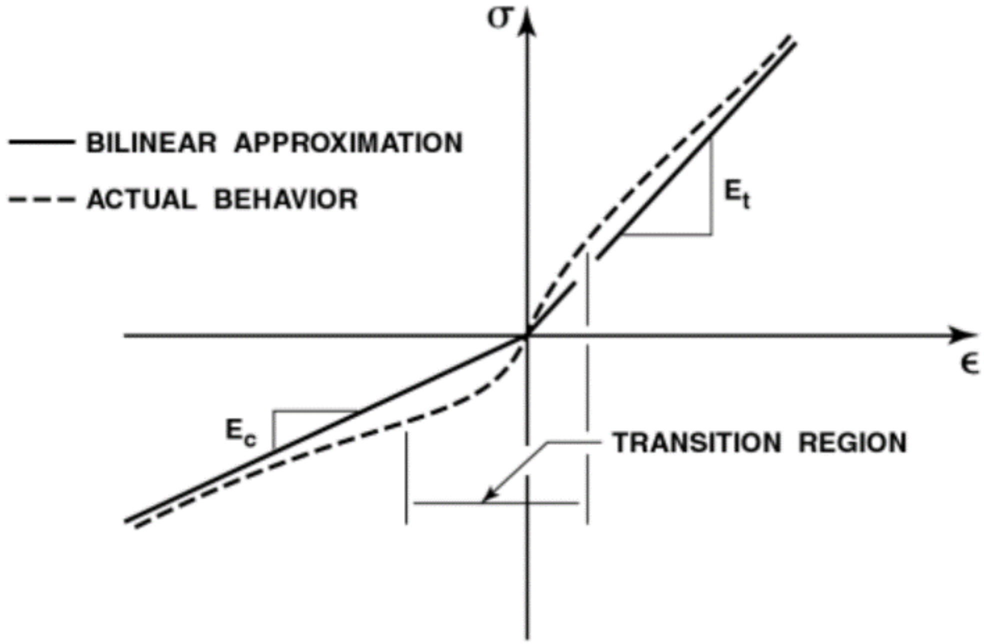
Six tests were made in each orientation on cold-rolled specimens. Curves: LT, longitudinal tensile; LC, longitudinal compressive; TT, transverse tensile; TC: transverse compressive. Elastic modulus: LT, 195.7 GPa; TT, 196.7 GPa; LC, 189.7 GPa; TC, 197.0 GPa. Yield strength (0.2%): LT, 359.6 MPa; TT, 383.1 MPa; LC, 295.8 MPa; TC, 380.2 MPa. Ultimate tensile strength: LT, 745 MPa; TT, 730 MPa. Composition: Fe-17Cr-6.5Mn-4.5Ni. UNS S20100

Source: P. Van Der Merwe and G.J Van Den Berg, The Advantages of Using Cr-Mn Steels Instead of Cr-Ni Steels in Cold-Formed Design, *High Manganese High Nitrogen Austenitic Steels,* R.A. Lula, Ed., Conf. Proc., 10–15 Oct 1987 (Cincinnati, OH) and 2–4 Nov 1992 (Chicago, IL), ASM International, 1992, p 129

Different deformation of materials under tension and compression (Bimodulus) is a result of different stress levels achieved at these conditions.

The modulus, strength, absorbed energy and failure stress may be different under different loading directions in such materials.

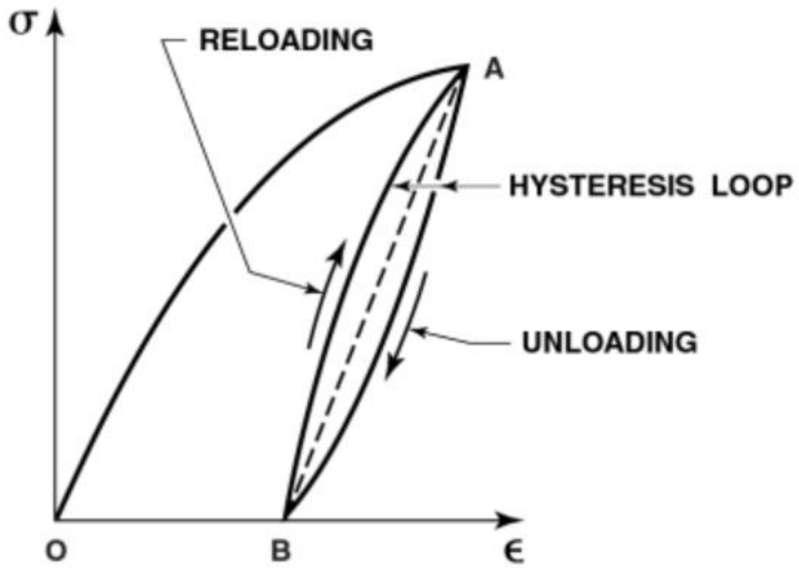
A suitable material model for bimodular materials that have mostly linear deformation is bilinear approximation



Unloading of materials after loading and the way they behave during unloading affects their deformation behavior upon further loading

If a material with linear stress-strain curve is loaded and unloaded, their curves coincide on the elastic region line

If a material with a nonlinear stress-strain curve is loaded to some stress level in the vicinity of the offset yield point, then the unloading behavior is concave upward and the subsequent loading curve is concave downward.



Typically there is a difference between the unloading curve and the reloading curve, a hysteresis loop is seen that indicates that energy is dissipated.

The dashed line is a useful idealization. Hysteresis loop is omitted and a simpler model is possible.

If after being loaded and unloaded, the material is loaded again, the new curve will rise parallel to the initial curve until it almost reaches point C and then connect with the curved portion of the original diagram.

Note that the straight elastic portion of the new loading curve is longer than the initial due to strain hardening.

The elastic limit has increased but the ductility decreased since the point of rupture is not changed.

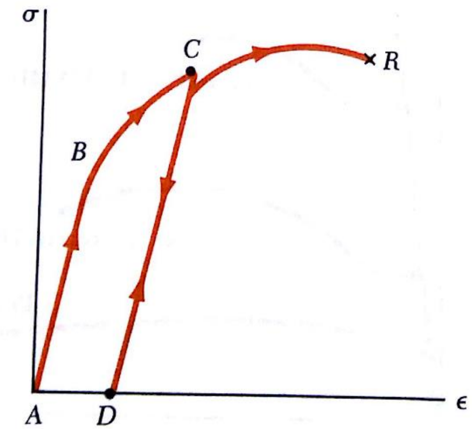
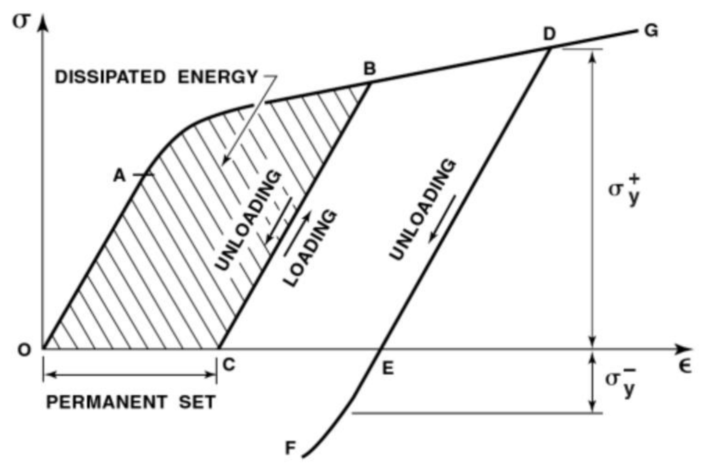


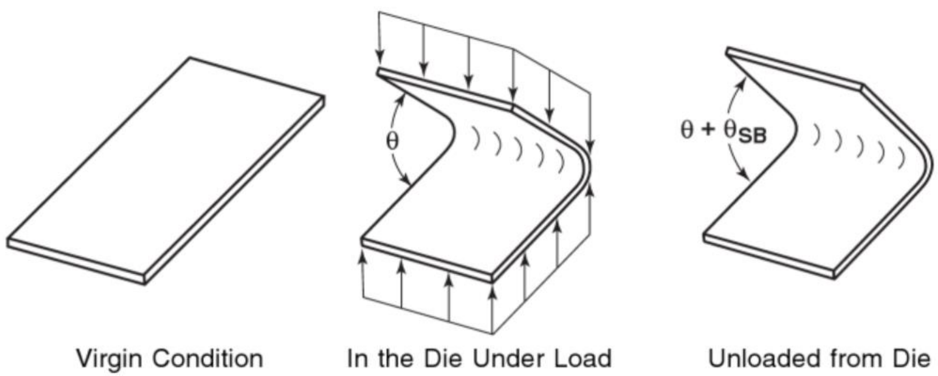
Fig. 2.17

An idealized loading and unloading behavior is reflected by a material with an initially linear stress-strain curve, followed by some nonlinearity which then becomes linear for a large portion of the curve.



The loading and unloading cycle produces some strain hardening, permanent deformation and an associated springback effect

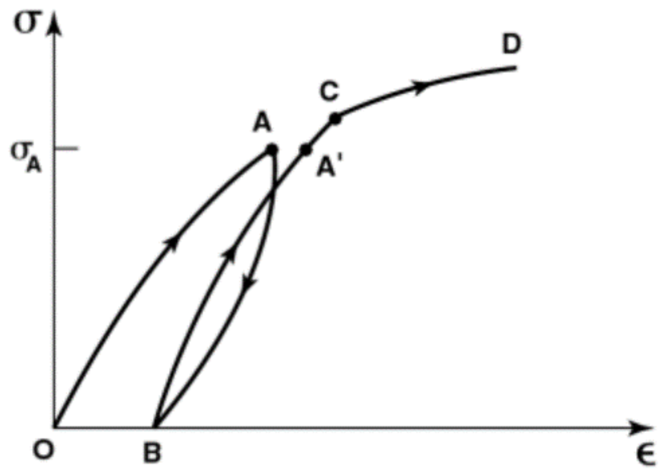
A body that is plastically deformed and then released, undeforms or springs back toward its original shape. Consider bending of a metal plate between a male and female die or through offset rollers:



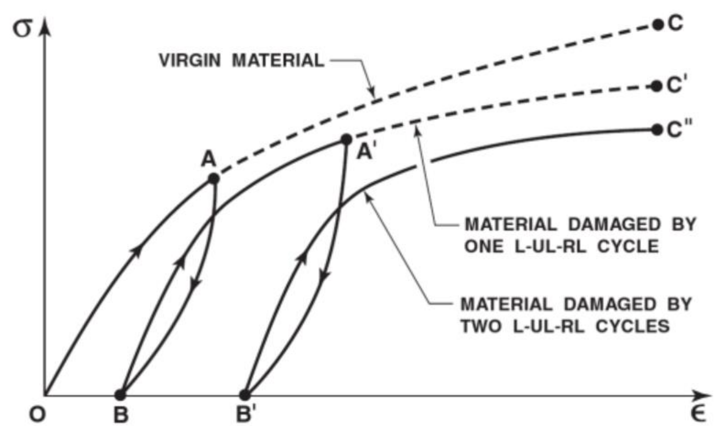
The reason for the springback is the elastic strain that is recovered during unloading

In reality the reloading curve does not join the unloading curve at the maximum stress level before unloading. Instead a larger strain is seen than the original curve at the same stress

The reason is that a cycle of loading, unloading and reloading causes irreversible damage to the material



The loading history affects both material microstructure and the resultant stress-strain curve
Progressive lowering of the stress-strain curves after unload-reload cycles from the virgin curve is called progressive damage

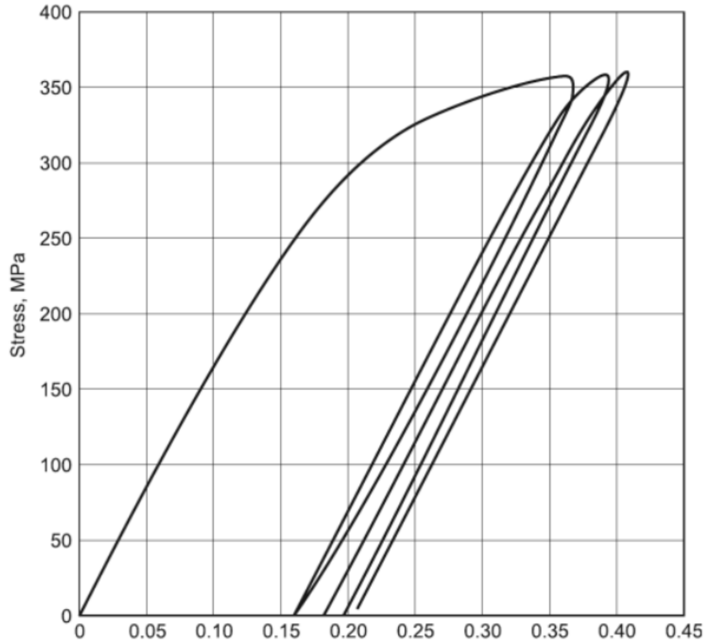


Presence of such loading history in the material complicates modeling material behavior

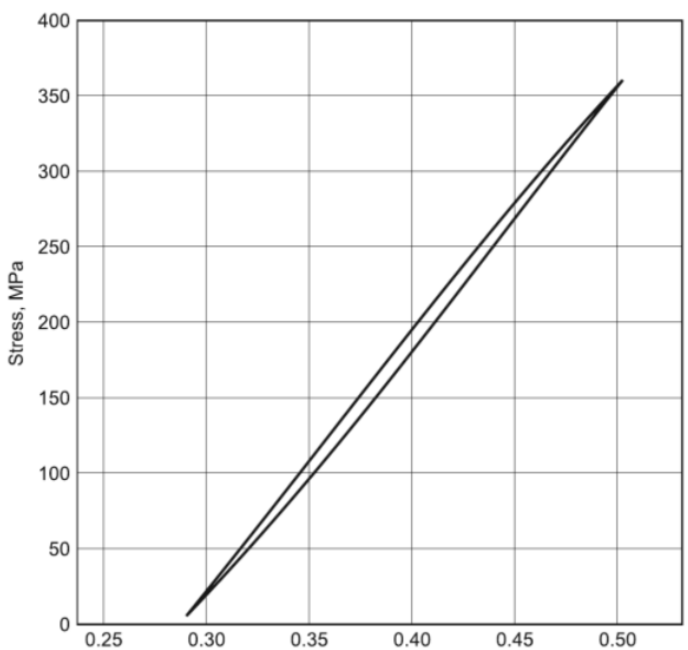
CI.035 Pearlitic nodular ductile iron casting, tensile stress-strain curves

Test direction: longitudinal. (a) Beginning of cycling in tension to 350 MPa. (b) Behavior of same sample after 128 cycles to 350 MPa. 0.2% proof stress = 358 MPa; ultimate tensile strength = 659 MPa. Composition: Fe-3.42C-2.11Si-0.31Mn-0.014S-0.007P-0.061Mg

Source: G.N.J. Gilbert and M.J.D. Frier, "The Stress/Strain Properties of a Pearlitic and a Nodular Cast Iron Cyclically Loaded between Equal and Opposite Strain Limits in Tension and Compression," Report 1579, British Cast Iron Research Association (BCIRA), 1984



(a)



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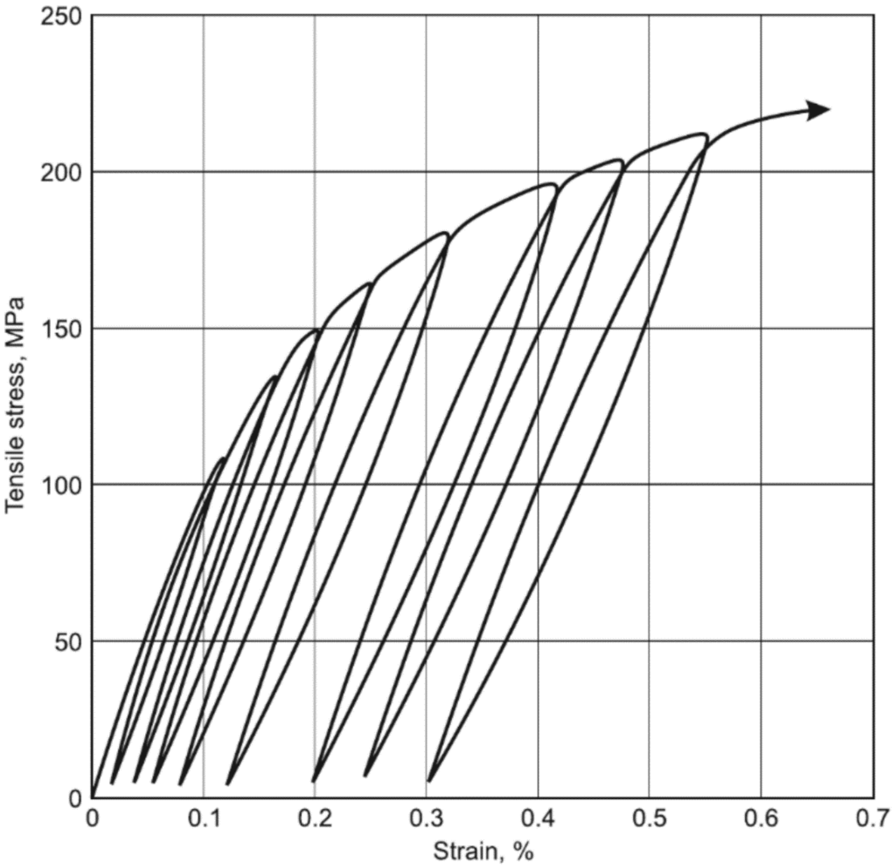
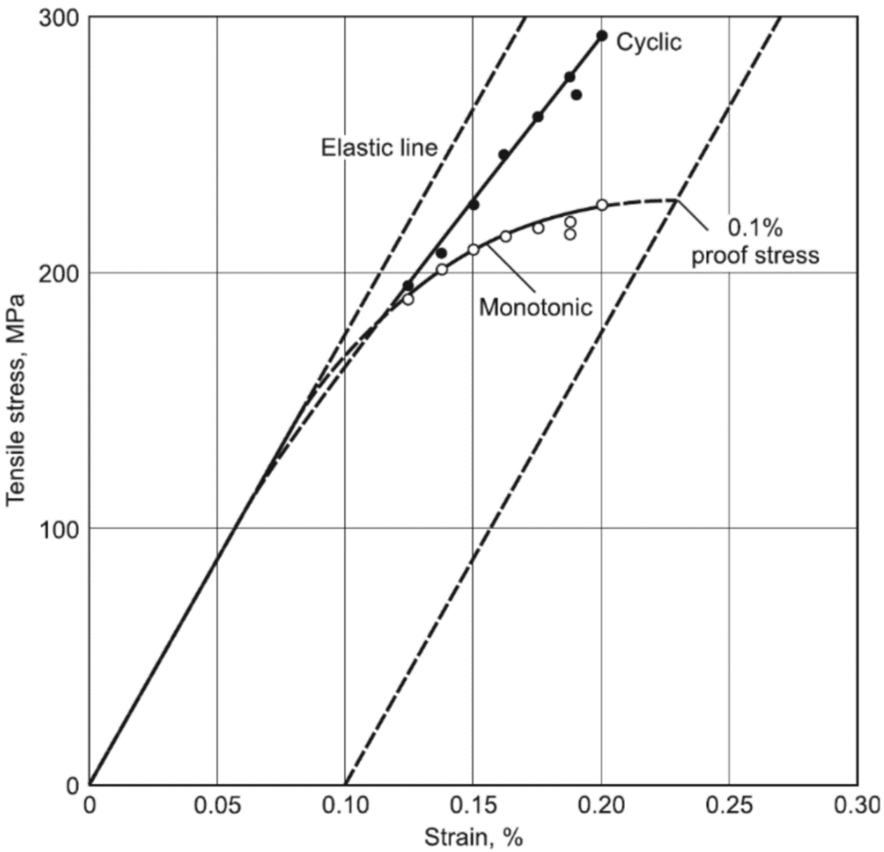


CI.052 Flake graphite, gray iron casting, tensile stress-strain curves with cyclic loading to increasing stress levels

Ultimate strength = 230 MPa. Permanent deformation increases with increasing stress levels.

Source: "Stress/Strain Behaviour of Flake Graphite Cast Irons," Broadsheet 157-1, British Cast Iron Research Association (BCIRA), 1977



CI.027 Ferritic nodular ductile iron casting, tensile monotonic and cyclic stress-strain curves

Curves based on the first cycle of loading and cycle tests carried out at less than 0.1% strain. The stress values are raised by strain hardening. Modulus of elasticity = 177 GPa. Composition: Fe-3.51C-2.07Si-0.32Mn-0.022S-0.017P-0.046Mg

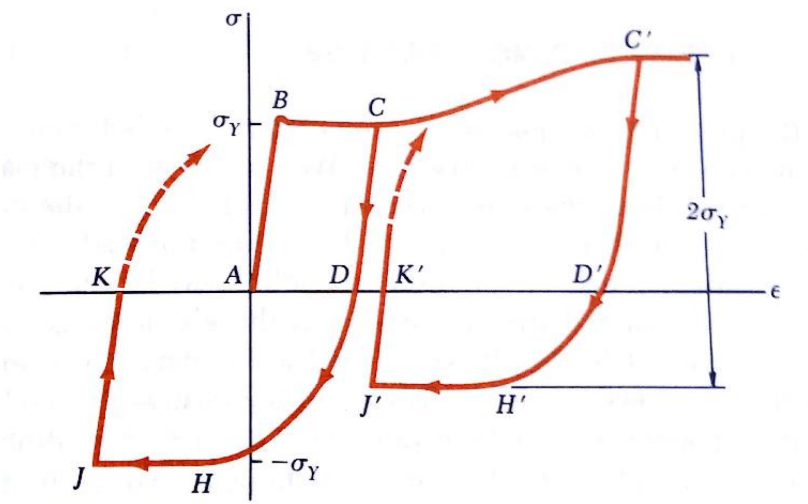
Source: G.N.J. Gilbert, "The Stress/Strain Properties and Fatigue Properties of a Ferritic and a Pearlitic Nodular Cast Iron Tested under Strain Control," Report 1586, British Cast Iron Research Association (BCIRA), 1984

If the second loading was done in compression instead of tension, an interesting behavior is observed for ductile materials

A compressive load is applied after the initial tensile load is removed at point D, compressing the material until the negative yield strength along a curved portion DHJ of the diagram

When the compressive load is removed at point J the stress returns to zero with an equal slope to the elastic region.

If the initial tension is large enough to cause strain hardening until point C’, the second compressive stress reaches its maximum value at H’ where material yields. While the maximum value of the compressive stress is less than the yield stress, the total change is equal to twice the yield strength of the material.



If the direction of the load is changed from tension to compression when we load and then unload a material, the yield stress changes due to the Bauschinger effect.

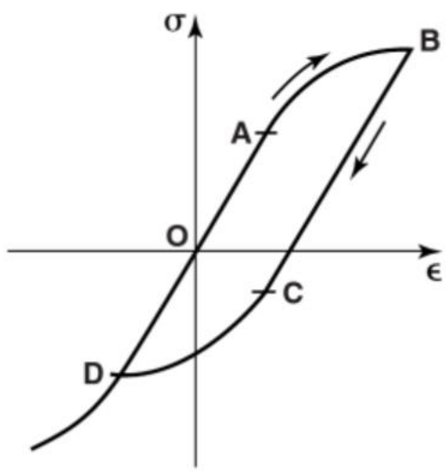


Figure 2-35 Bauschinger Effect for a General Linear-Nonlinear Curve

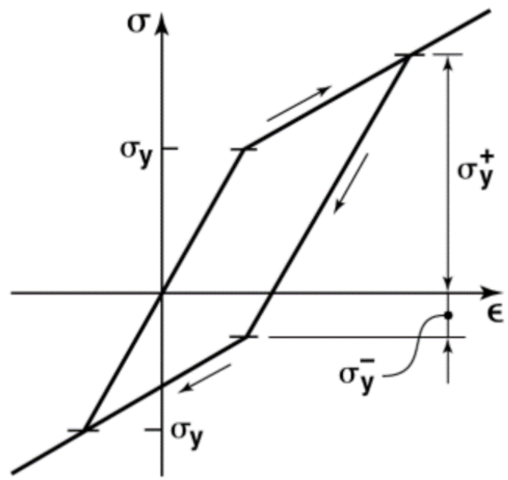


Figure 2-36 Bauschinger Effect for a Linear Strain-Hardening Curve

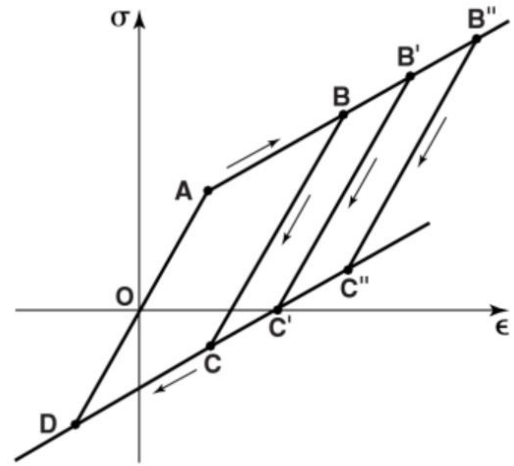
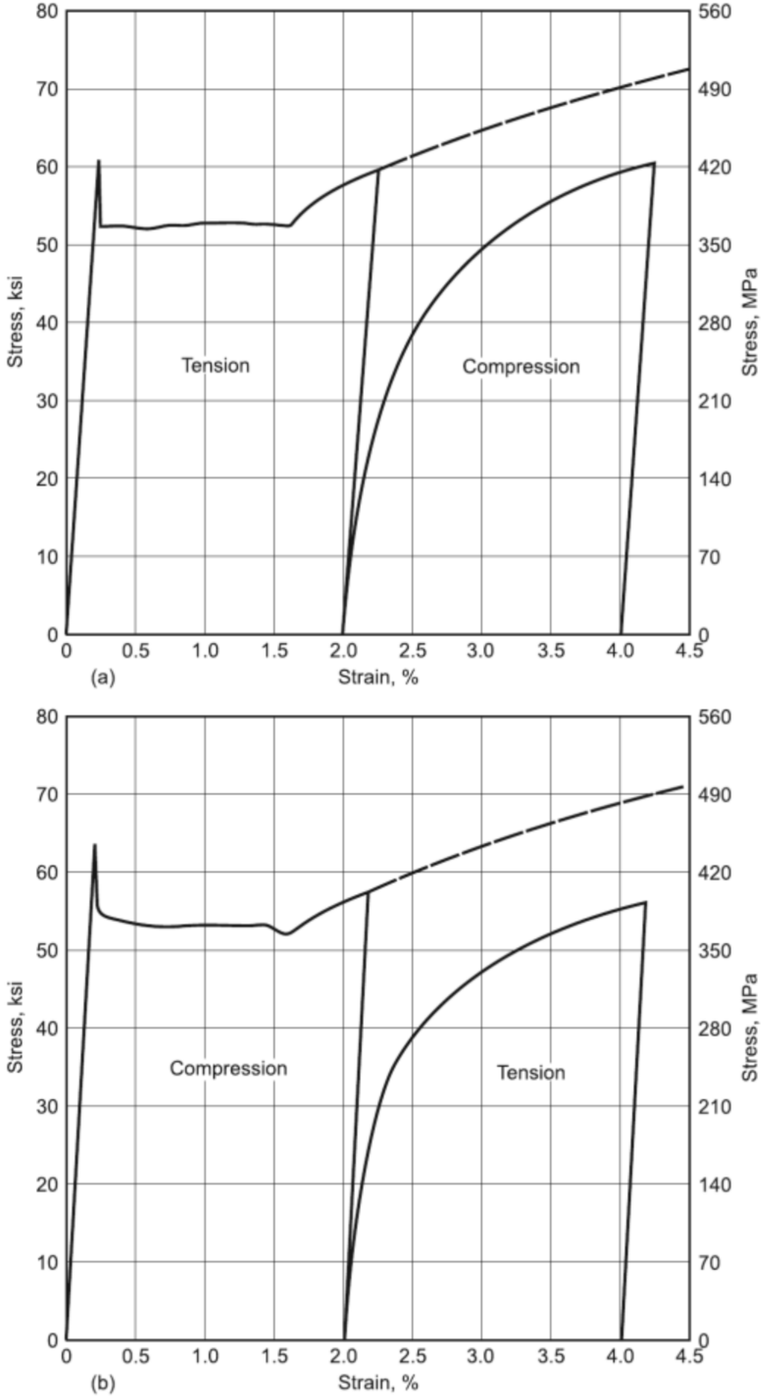


Figure 2-37 Bauschinger Effect for a Linear Strain-Hardening Curve with Multiple Load-Unload Cycles

This behavior is observed for many metals. The same strain hardening can be produced by a single loading or cyclic loadings.

In any case

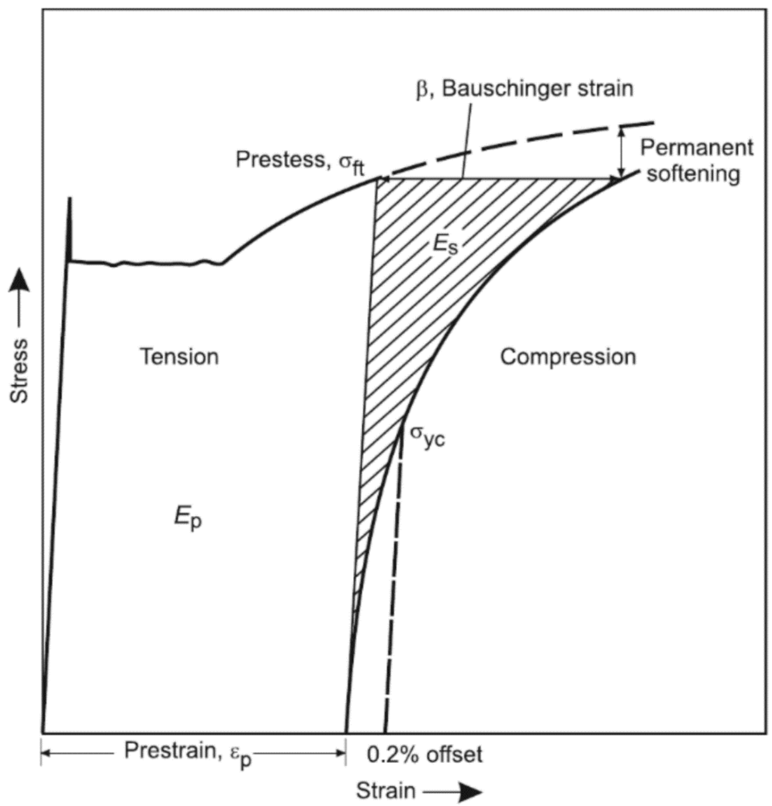
$$\sigma_y{}^{+} + \sigma_y{}^{-} = 2\sigma_y$$



CS.026 1020 carbon steel, true stress-strain curves

(a) Bauschinger effect shown for test sequence of tension to 2% strain followed by compression of another 2%. (b) The sequence is compression-tension. Tested at 25 °C. Composition: Fe-0.21C-0.64Mn-0.030S-0.018P-0.23Si-0.007N. UNS G10200

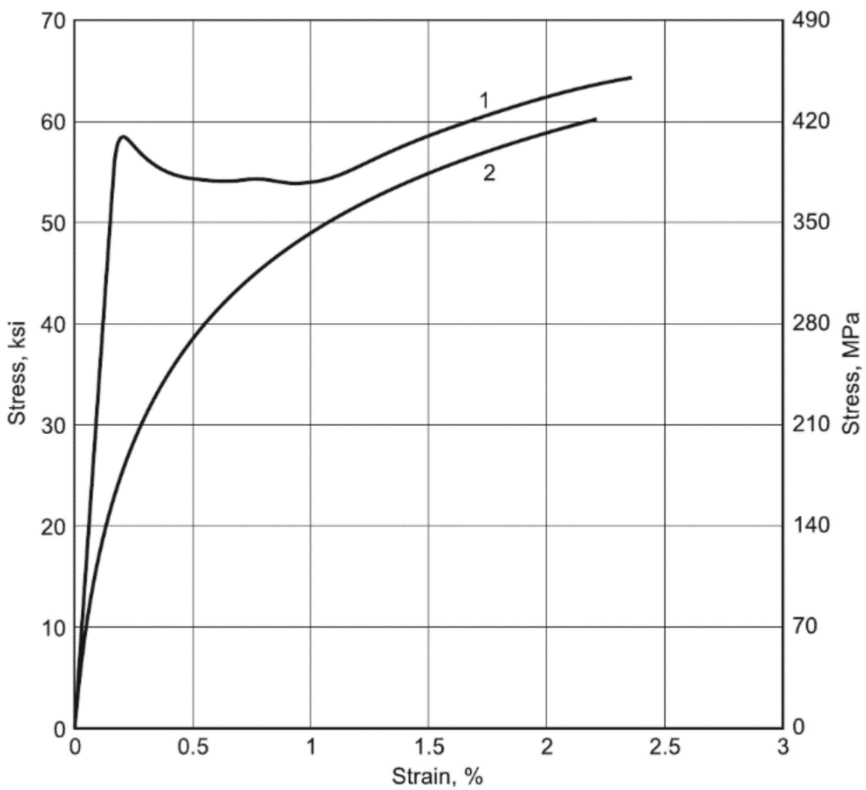
Source: C.-C. Li, J.D. Flasck, J.A. Yaker, and W.C. Leslie, On Minimizing the Bauschinger Effect in Steels by Dynamic Strain Aging, *Metall. Trans. A,* Jan 1978, p 86



CS.025 Carbon steel, Bauschinger effect on stress-strain curves

The elastic limit of a metal is lowered after reverse loading. The area E_p is the energy expended in prestrain, and E_s is the energy saved in reverse loading.

Source: C.-C. Li, J.D. Flasck, J.A. Yaker, and W.C. Leslie, On Minimizing the Bauschinger Effect in Steels by Dynamic Strain Aging, *Metall. Trans. A,* Jan 1978, p 86



CS.027 1020 carbon steel, true stress-strain curves

Curve 1: specimen is prestrained in tension at 250 °C to 2% strain and tested in compression at room temperature. Curve 2: the specimen is prestrained in tension at room temperature to 2% strain and tested in compression at room temperature. The Bauschinger effect is reduced. Composition: Fe-0.21C-0.64Mn-0.030S-0.018P-0.23Si-0.007N. UNS G10200

Source: C.-C. Li, J.D. Flasck, J.A. Yaker, and W.C. Leslie, On Minimizing the Bauschinger Effect in Steels by Dynamic Strain Aging, *Metall. Trans. A,* Jan 1978, p 88

Fatigue

Although the elastic region of the stress-strain curve is a safe zone against plastic deformation, repeating the load many times (millions) will cause rupture at a stress much lower than the yield strength of the material, hence fatigue.

The stress-cycle curve for steel shows that relatively few cycles are enough to cause rupture if the applied maximum stress is high.

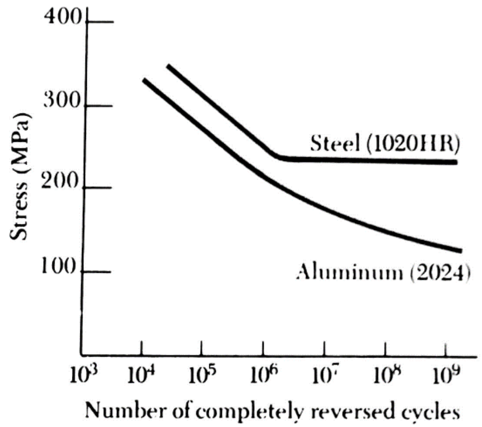


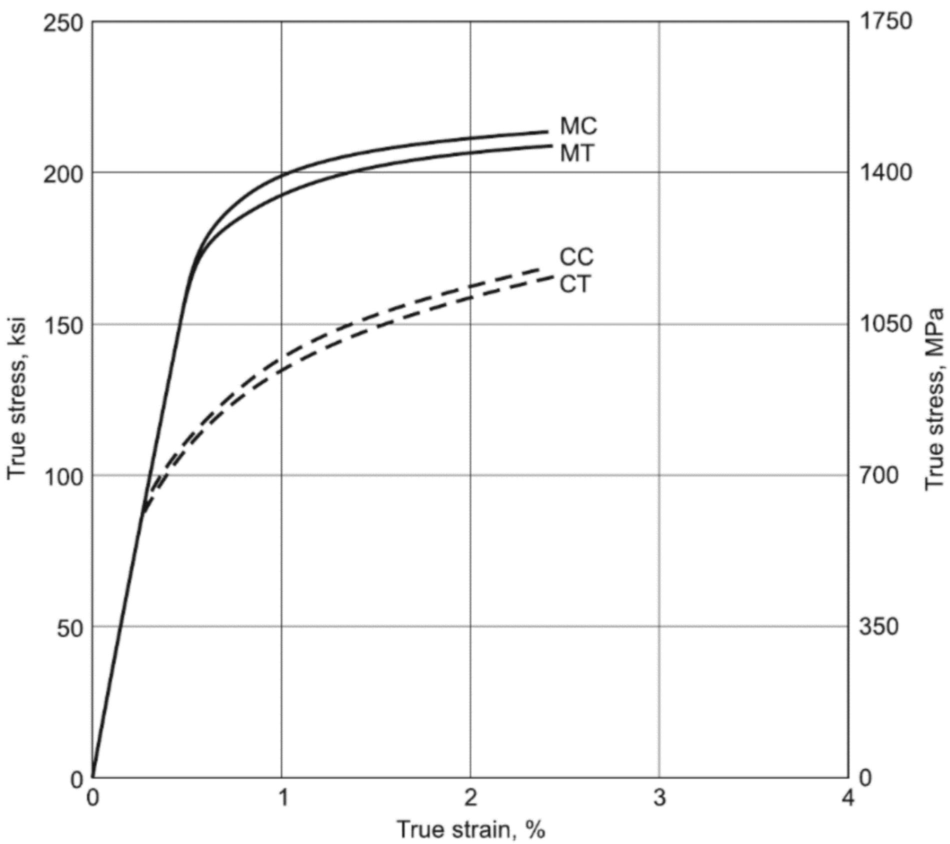
Fig. 2.19

As the magnitude of stress is reduced, the number of cycles to rupture increases. For steel a stress limit is reached called the endurance limit, below which it will not rupture even for infinitely many cycles.

For non-ferrous metals like aluminum, the stress at failure continues to decrease as the number of loading cycles is increased. The stress around 500 million cycles is called the fatigue limit.

The mechanism of fatigue is slow propagation of a crack that initiate at an imperfection with each cycle

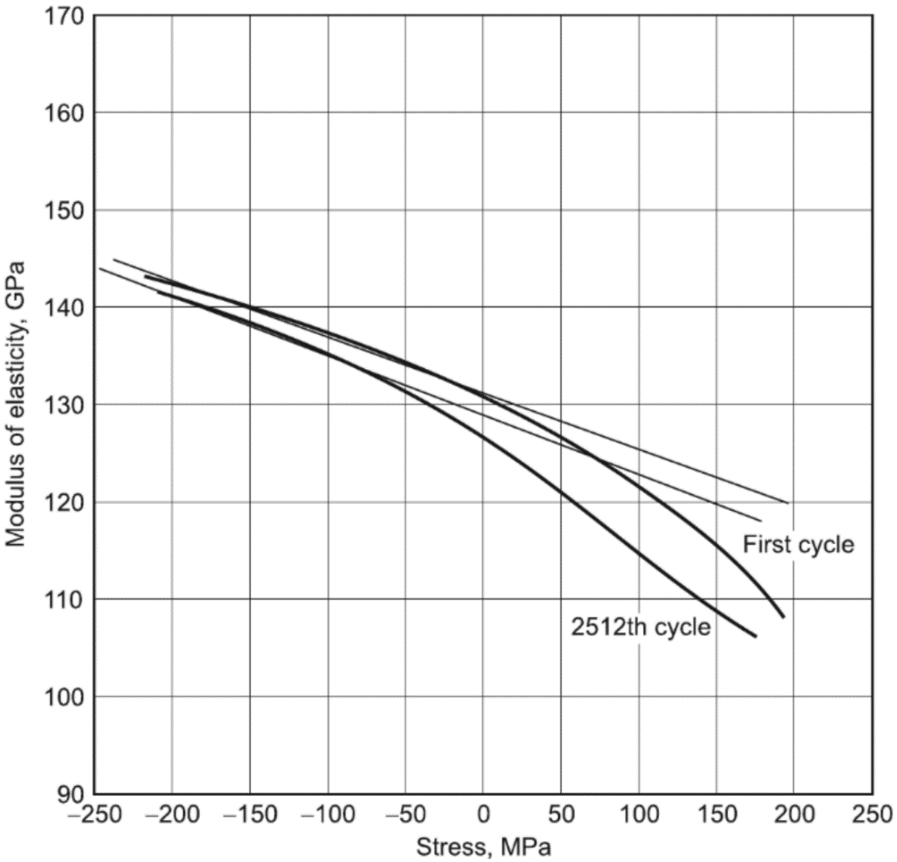
Sudden brittle fracture occurs when the amount of undamaged material is insufficient to carry the maximum load



AS.014 4140 chromium-molybdenum alloy steel bar, monotonic and cyclic true stress-strain curves

Heat treatment: austenitized 999 °C (1830 °F), 1 h, oil quenched, tempered 399 °C (750 °F), 1 h, water quenched. Gage section size = 5.08 mm diam × 7.62 mm long (0.2 in. diam × 0.3 in. long). Strain rate = 0.5/min. Test condition: MT, monotonic tension; MC, monotonic compression; CT, cyclic tension; CC, cyclic compression. Composition: Fe-0.4C-1Cr-0.2Mo. UNS G41400

Source: P.N. Thielen, M.F. Fine, and R.A. Fournelle, Cyclic Stress Strain Relations and Strain-Controlled Fatigue of 4140 Steel, *Acta Metall.,* Vol 24 (No. 1), Jan 1976, p 1–10. As published in *Aerospace Structural Metals Handbook,* Vol 1, Code 1203, CINDAS/USAF CRDA Handbooks Operation, Purdue University, 1995, p 18



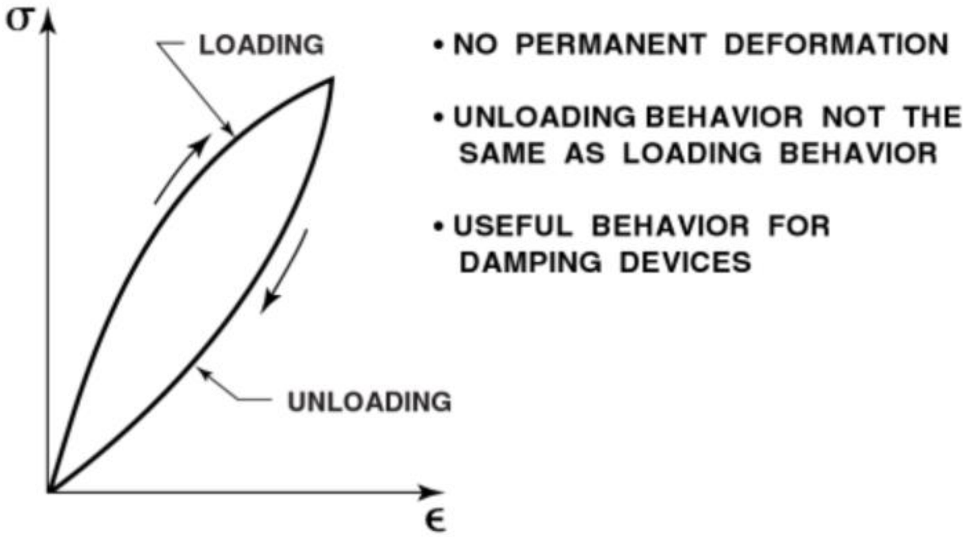
CI.058 Gray iron casting, modulus of elasticity-stress curves

Modulus of elasticity (E) for compression of first and 2512th cycle. At maximum compressive stress (0.0020 strain controlled) first cycle, $E = 144.95$ GPa; 2512th cycle, $E = 144.20$ GPa

Source: G.N.J. Gilbert, "The Cyclic Stress/Strain Properties and Fatigue Properties of a Flake Graphite Cast Iron Tested under Strain Control—A Detailed Study," Report 1621, British Cast Iron Research Association (BCIRA), 1985

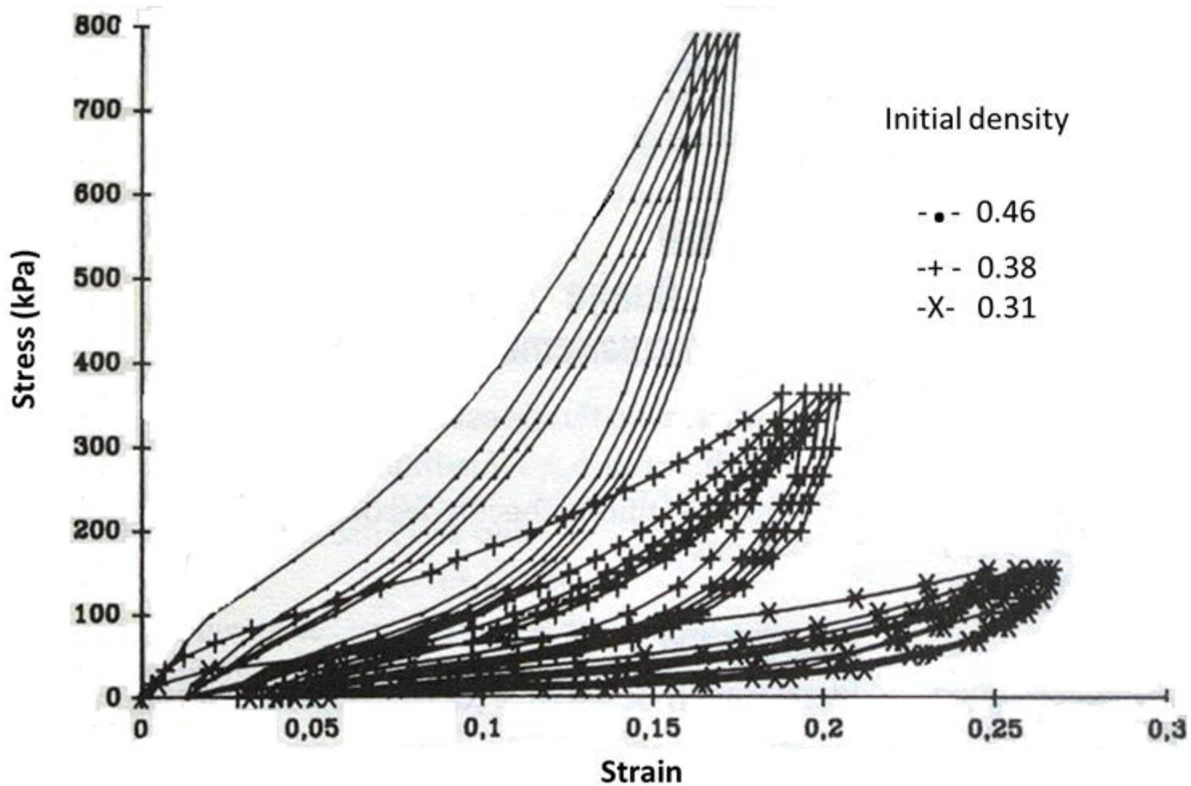
Consider another material behavior phenomenon called anelastic behavior.

When we load and unload an anelastic material, we have no permanent deformation so all the deformation energy is dissipated as heat.



This behavior is different than elastic behavior because the loading and unloading curves are different, plastic.

A few materials exhibit this behavior and are used as shock absorbers or dampers



Stress (kPa)
800
700
600
500
400
300
200
100
0
0
0,05
0,1
0,15
0,2
0,25
0,3
Strain
Initial density
-•- 0.46
-+ - 0.38
-X- 0.31

Overall the factors that must be taken into account when modeling nonlinear materials are:

- Degree or form of nonlinearity
- Effect of temperature change
- Effect of moisture content change
- Unloading
- History of deformation
- Tension versus compression behavior
- Degree of anisotropy

Idealizations of stress-strain behavior are the key to practical engineering analysis of deformation processes.

All of the above factors complicate the material model to use as an idealization of the actual deformation

A material model more complex than necessary should be avoided as it may not be possible to solve.